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Partial waves of baryon-antibaryon in three-body B meson decay

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Abstract

The conspicuous threshold enhancement has been observed in the baryon-antibaryon subchannels of many three-body B decay modes. By examining the partial waves of baryon-antibaryon, we first show for $B^{\pm} \to p\overline{p}K^{\pm}$ that the $pK^{\pm}$ angular correlation rules out dominance of a single $p\overline{p}$ partial wave for the $p\overline{p}$ enhancement, for instance, the resonance hypothesis or the strong final-state interaction in a single channel. The measured $pK^{\pm}$ angular correlation turns out to be opposite to the theoretical expectation of a simple short-distance picture. We study the origin of this reversed angular correlation in the context of the $p\overline{p}$ partial waves and argue that $N\overline{N}$ bound states may be the cause of this sign reversal. Dependence of the angular correlation on the $p\overline{p}$ invariant mass is important to probe the underlying issue from the experimental side.

I. INTRODUCTION

In the baryonic B decay the three-body modes dominate over the two-body modes. Furthermore, in the three-body decay, the baryon-antibaryon pair is copiously produced at small invariant mass near the threshold[1–5]. Various theoretical ideas[6–10], some kinematical and others dynamical, were proposed for this threshold enhancement. A simple short-distance (SD) argument can explain qualitatively both the dominance of three-body modes and the threshold enhancement of baryon-antibaryon: To produce a baryon and an antibaryon in the two-body decay (Fig. 1a), one energetic $q\overline{q}$ pair must be emitted back to back by a gluon so that the gluon emitting the $q\overline{q}$ pair is highly off mass shell. The hard off-shell gluon suppresses two-body decay amplitudes by the power of α_s/t, where t is the four-momentum square transferred through the gluon. In the three-body decay with an additional meson (Fig. 1b), a baryon-antibaryon pair can be emitted collinearly against the energetic boson in the final state. In this configuration a quark and an antiquark are emitted by a gluon nearly in the same direction so that the gluon is close to the mass shell and the short-distance suppression does not occur. Consequently the $p\overline{p}$ of small invariant mass is strongly favoured.

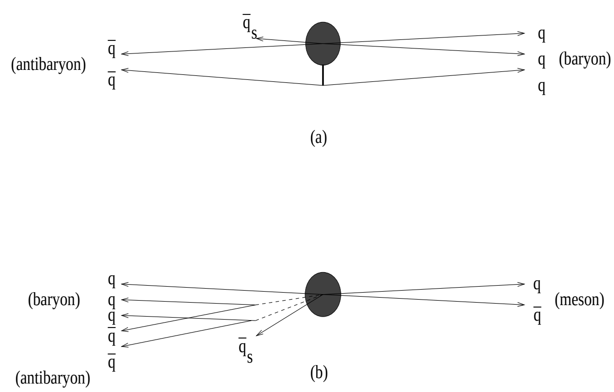

FIG. 1: Short-distance picture in quarks and antiquarks for (a) two-body baryonic decay and (b) three-body baryonic decay. In the two-body decay (a) the fat virtual gluon (the thick vertical solid line) must split into $q\overline{q}$ while in the three-body decay (b) nearly on-shell gluons (broken lines) turn into $q\overline{q}$. The slow spectator antiquark is denoted by the short line $\overline{q}_s$.

In addition to threshold enhancement, the angular correlation was measured between the final proton or antiproton and the boson in some modes, most clearly in $B^{\pm} \to p\overline{p}K^{\pm}$[11]. Then an intriguing puzzle[10] has emerged in the preceding SD picture: In that picture, the antiproton momentum should point more likely to the direction of the K^- momentum in the $p\overline{p}$ rest frame of $B^-(b\overline{u})$ decay. That is, the proton should tend to move away from K^- in this frame. The reason is that the antiproton $\overline{p}$ picks up the slow spectator $\overline{u}$-quark and therefore its momentum is smaller on average than that of the proton p in the B^- rest frame. By boosting the B^- rest frame to the $p\overline{p}$ rest frame, we reach this conclusion.

However, the Belle Collaboration showed exactly the opposite[11]; it is the proton that is emitted along K^- in the $p\overline{p}$ rest frame. Belle selected the threshold events by making a cut in the $p\overline{p}$ invariance mass $m_{p\overline{p}}(< 2.85$ GeV$)$, but did not give the angular correlation as a function of $m_{p\overline{p}}$ for the selected events. Meanwhile BaBar gave a Dalitz plot of $p\overline{p}K^-$[3] from which one can read the same trend as Belle's angular dependence.

Rosner[8] argued qualitatively in terms of quark diagrams[12] and predicted this angular correlation with baryon production through diquarks. But the argument does not seem to work for all baryonic modes in its simple form. Cheng and others[10] computed the decay amplitudes in the pole model with factorization, leaving out inelastic form factor terms[9, 10]. Their result does not lead to the correct angular correlation in the case of $B^- \to p\overline{p}K^-$. The simple SD picture presented at the beginning is successful in the angular correlation of most three-body baryonic modes, *e.g.,* $B^- \to \Lambda\overline{p}\gamma$, but fails notably for $B^- \to p\overline{p}K^-$. Its failure suggests us importance of long-distance (LD) effects somewhere in the decay process. Indeed, the fragmentation by quark diagram and the pole model both contain some of the LD effects in very different ways. In this paper we take a close look at this angular correlation of $B^\pm \to p\overline{p}K^\pm$ from the viewpoint of partial waves in general and try to resurrect the simple SD picture by incorporating an appropriate LD effect in it.

In our proposed analysis we first examine the partial-wave content of $p\overline{p}$ in $B^- \to p\overline{p}K^-$ (and its conjugate) and conclude purely kinematically that the $p\overline{p}$ enhancement cannot be a broad resonance. For the same reason we rule out strong final-state interaction (FSI) in a single $p\overline{p}$ partial wave as a cause of the enhancement. We shall observe that reversal of the angular correlation occurs if some LD effect flips relative signs of partial-wave decay amplitudes. Such sign flip may indeed occur if $N\overline{N}$ bound states exist in right channels. The recently discovered state $X(1835)$[13] is a good candidate that may be responsible for the sign flip. If $X(1835)$ should be an $N\overline{N}$ bound state, we expect a similar bound state in other channels from our reasoning of the sign flip.

II. $p\overline{p}$ PARTIAL WAVES IN $B^- \to p\overline{p}K^-$

We study the angular correlation between the proton momentum and the kaon momentum in the rest frame of $p\overline{p}$ by choosing the z-axis along the K^- momentum. (Fig. 2)

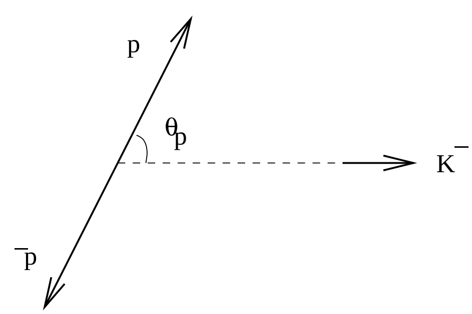

FIG. 2: The pK^- angular correlation in the $p\overline{p}$ rest frame.

Since B^- meson and K^- meson are spinless and their momenta are both along the z-direction in the $p\overline{p}$ rest frame, the z-component of total angular momentum is zero for $p\overline{p}$ by $\Delta J_z = 0$ in this frame. Following the standard helicity formalism[14], we can describe the angular dependence of the helicity decay amplitudes with Wigner's d-functions [15] as

$$A(B^- \to p\overline{p}K^-) = \sum_J A^{JK^-}_{\lambda_p\lambda_{\overline{p}};0} d^J_{0\lambda}(\theta_p) e^{-i\lambda\phi_p}, \quad (\lambda = \lambda_p - \lambda_{\overline{p}}), \tag{1}$$

where λ_p and $\lambda_{\overline{p}}$ are the helicities of p and $\overline{p}$ in the $p\overline{p}$ rest frame, (θ_p, ϕ_p) are the angles of the proton momentum in this frame (Fig. 2), and $A^{JK^-}_{\lambda_p\lambda_{\overline{p}};0}$ is a function of the $p\overline{p}$ invariant mass $m_{p\overline{p}}$. Since experiment does not measure helicity of proton nor antiproton but sums over all helicity states in what follows, the differential decay rates are ϕ_p independent. Therefore we need not specify the direction of $\phi_p = 0$ in our case.[1] Squaring the amplitude and summing over the $p\overline{p}$ helicities λ_p and $\lambda_{\overline{p}}$, we obtain the differential decay rate:

$$\frac{d\Gamma(\theta_p)}{dm_{p\overline{p}}d\Omega_p}\bigg|_{B^-\to p\overline{p}K^-} = \Gamma_0 \sum_{\lambda_p\lambda_{\overline{p}}}\Big|\sum_J A^{JK^-}_{\lambda_p\lambda_{\overline{p}};0} d^J_{0\lambda}(\theta_p)\Big|^2, \tag{2}$$

where Γ_0 includes kinematical factors that depend on $m_{p\overline{p}}$. If we make the usual assumption that the strong penguin interaction dominates in the decay $\overline{B} \to p\overline{p}\overline{K}$, the CP-violating phases drop out of the decay rate. Under parity reflection the angle θ_p remains unchanged, while under charge conjugation the angle θ_p turns into $\pi - \theta_p$ of $B^+ \to p\overline{p}K^+$ because of the interchange $p \leftrightarrow \overline{p}$ and $K^- \leftrightarrow K^+$. Therefore,

$$\frac{d\Gamma(\theta_p)}{dm_{p\overline{p}}d\Omega_p}\bigg|_{B^+\to p\overline{p}K^+} = \frac{d\Gamma(\pi-\theta_p)}{dm_{p\overline{p}}d\Omega_p}\bigg|_{B^-\to p\overline{p}K^-}. \tag{3}$$

We shall be able to use this equality as a test of the penguin dominance. The corresponding relation holds between $B^0 \to p\overline{p}K^0$ and $\overline{B}^0 \to p\overline{p}\overline{K}^0$.

At this stage we can prove that the $p\overline{p}$ enhancement is not a resonance, for instance, a glueball[7]: The Wigner functions $d^J_{0\lambda}(\theta)$, which are proportional to the associated Legendre functions, possess a special symmetry property under $\theta \leftrightarrow \pi - \theta$[15],

$$d^J_{0\lambda}(\pi-\theta) = (-1)^{J+\lambda} d^J_{0\lambda}(\theta) \quad \to \quad |d^J_{0\lambda}(\pi-\theta)|^2 = |d^J_{0\lambda}(\theta)|^2. \tag{4}$$

If the $p\overline{p}$ pair is produced entirely through a resonance, only the term of the resonance spin J contributes in Eq. (2) without sum over J. Since the function $|d^J_{0\lambda}(\theta)|^2$ is unchanged under $\theta \to \pi - \theta$ (*i.e.,* $\cos\theta \to -\cos\theta$), so is $d\Gamma/d\Omega_p$ in this case. However, experiment shows a pronounced asymmetry between two hemispheres of $\cos\theta > 0$ and $\cos\theta < 0$. (Fig. 3.) In terms of the forward-backward asymmetry parameter[11],

$$A \equiv (N_+ - N_-)/(N_+ + N_-) = 0.59^{+0.08}_{-0.07} \tag{5}$$

contrary to $A = 0$ in the case of a single J. Although interference between the resonant and nonresonant amplitudes of different J can produce some asymmetry in principle, such interference should be insignificant under the normal circumstance where the resonant amplitude acquires the phase $\frac{\pi}{2}$ through the resonant decay relative to the nonresonant amplitude: $\arg(A^J_{\rm res}A^{J'*}_{\rm non}) \simeq \pm\pi/2$. It is implicitly assumed here as usual that the nonresonant production amplitude dose not acquire a significant phase. If for some reason the large asymmetry of Fig.3 should be caused by the interference between the resonant and nonresonant amplitudes of different J's, the very small yield observed toward $\theta_p = \pi$ in Fig. 3 would mean nearly perfect destructive interference between them. In this case the nonresonant yield would have to be just as large as the resonant one. Therefore the marked asymmetry in the

[1] We would have to fix the $\phi_p = 0$ direction if a final particle spin is measured or if a final particle having spin undergoes a cascade decay and this decay angular correlation is measured.

angular correlation rules out convincingly the hypothesis of pure resonant production. If one attempts to explain the the enhancement by strong FSI in a single dominant partial-wave channel of $p\overline{p}$[16], one would likewise obtain $A \simeq 0$ for the angular correlation. To be consistent with the observed angular correlation, partial-wave amplitudes of even and odd J must coexist and almost maximally interfere. Our argument is very general and independent of dynamics up to this point. We now proceed to take dynamics into account.

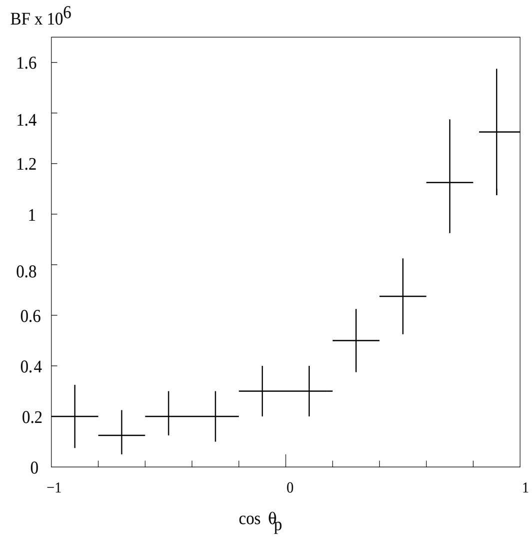

FIG. 3: The pK^- angular distribution in the $p\overline{p}$ rest frame [Ref. 10].

In the SD picture the spectator $\overline{u}$-quark of B^- enters the antiproton with two energetic antiquarks ($\overline{u}$ and $\overline{d}$) which are pair-produced nearly collinearly by two gluons. (Fig. 1b) Note that it is a color-suppressed process for the s-quark from $b \to sg^*$ of the strong penguin decay to form K^- by capturing the spectator $\overline{u}$.[2] In contrast, the proton consists of three energetic quarks; one from the primary decay interaction and two of pair-produced quarks. In the B^- rest frame, therefore, the proton recoils against K^- more energetically on average than the antiproton does. It means that in the $p\overline{p}$ rest frame, the proton tends to move away from K^- faster than the antiproton does. That is, $A < 0$ contrary to the measurement. This is the "angular correlation puzzle". No reasonable explanation has been given from the SD viewpoint. We must look for some LD interaction effect that has not been commonly appreciated.

The maximum of the $p\overline{p}$ enhancement occurs near $m_{p\overline{p}} = 2$ GeV in the BaBar data[3] and roughly $\leq$2.2 GeV in the Belle data[11]. The dominant relative orbital angular momenta of $p\overline{p}$ are expected to be s-wave and p-wave. The amount of d-wave is presumably small and higher waves are even smaller. The terms that contribute dominantly in Eq. (1) are therefore $J = 0$ ($^1S_0,^3P_0$), $J = 1$ ($^3S_1,^3P_1,^1P_1$), and $J = 2$ ($^3P_2$). The explicit forms of the relevant $d^J_{0\lambda}$ functions ($J \leq 2$, $\lambda = -1, 0, +1$) are[15]:

$$d^0_{00}(\theta) \ = \ 1, \ \ d^1_{00}(\theta) = \cos\theta, \ \ d^2_{00}(\theta) = (3\cos^2\theta - 1)/2,$$

[2] We leave out the radiative penguin interaction here since it does not affect the leading behavior due to the strong penguin interaction.

$$d^1_{0\pm1}(\theta) \;=\; \mp\sqrt{1/2}\sin\theta, \;\; d^2_{0\pm1}(\theta) = \mp\sqrt{3/2}\sin\theta\cos\theta, \tag{6}$$

It is convenient to rearrange the helicity decay amplitudes $A^J_{\lambda_p\lambda_{\overline{p}};0}$ with the spectroscopic notation into $A(^{2S+1}L_J;\lambda_p-\lambda_{\overline{p}})$. When only s-waves and p-waves are retained, the helicity amplitudes of definite isospin I for $N\overline{N}$ can be written as

$$\begin{aligned}
A^{0I}_{\pm\pm;0} &= \pm A^I(^1S_0,0) + A^I(^3P_0,0),\\
A^{1I}_{\pm\pm;0} &= A^I(^3S_1,0) \pm A^I(^1P_1,0),\\
A^{1I}_{\pm\mp;0} &= \sqrt{2}A^I(^3S_1;\pm1) \pm A^I(^3P_1;\pm1),\\
A^{2I}_{\pm\pm;0} &= A^I(^3P_2;0),\\
A^{2I}_{\pm\mp;0} &= \sqrt{3/2}A^I(^3P_2;\pm1),
\end{aligned} \tag{7}$$

where we have denoted the helicity indices $\lambda_p,\lambda_{\overline{p}} = \pm1/2$ simply by $\pm$. We shall use this notation hereafter. The normalization of the amplitudes is arbitrary for the decay amplitudes which have no unitarity constraint. Since $\Delta I = 0$ for the strong penguin decay, the decay amplitudes for the charge eigenstates of $\overline{B}$ and $\overline{K}$ are given by the decay amplitudes $A^I(^{2S+1}L_J,\lambda)$ of definite $N\overline{N}$ isospin as

$$\begin{aligned}
A^{K^-}(^{2S+1}L_J,\lambda) &= \sqrt{1/2}\Big[A^1(^{2S+1}L_J,\lambda) - A^0(^{2S+1}L_J,\lambda)\Big],\\
A^{\overline{K}^0}(^{2S+1}L_J,\lambda) &= \sqrt{1/2}\Big[A^1(^{2S+1}L_J,\lambda) + A^0(^{2S+1}L_J,\lambda)\Big].
\end{aligned} \tag{8}$$

Combining Eqs. (7) and (8), we obtain the decay amplitudes as functions of θ_p, ϕ_p and $m_{p\overline{p}}$. The decay amplitudes for $B^+/B^0 \to p\overline{p}K^+/p\overline{p}K^0$ are obtained from those of $B^-/\overline{B}^0 \to p\overline{p}K^-/p\overline{p}\overline{K}^0$ with the interchange $\lambda_p \leftrightarrow \lambda_{\overline{p}}$ followed by $\theta_p \to \pi-\theta_p$ and $\phi_p \to \pi+\phi_p$ up to the overall CP phase factor of the penguin decay.

We are now able to write the complete differential decay rate for $B^- \to p\overline{p}K^-$ with $p\overline{p}$ in s and p-waves in the notation of $^{2S+1}L_J$:

$$\begin{aligned}
\frac{d\Gamma}{dm_{p\overline{p}}d\Omega_p}\bigg|_{B^-\to p\overline{p}K^-} &= \Gamma_0\Big|\Big(A^{K^-}(^1S_0,0) + A^{K^-}(^3P_0,0)\Big) + \Big(A^{K^-}(^3S_1,0) + A^{K^-}(^1P_1,0)\Big)\cos\theta_p\\
&+ A^{K^-}(^3P_2,0)(3\cos^2\theta_p - 1)/2\Big|^2\\
&+ \Big|\Big(-A^{K^-}(^1S_0,0) + A^{K^-}(^3P_0,0)\Big) + \Big(A^{K^-}(^3S_1,0) - A^{K^-}(^1P_1,0)\Big)\cos\theta_p\\
&+ A^{K^-}(^3P_2,0)(3\cos^2\theta_p - 1)/2\Big|^2\\
&+ \Big|\Big(A^{K^-}(^3S_1,1) + \sqrt{1/2}A^{K^-}(^3P_1,1)\Big)\sin\theta_p\\
&+ A^{K^-}(^3P_2,1)(3\sin\theta_p\cos\theta_p)/2\Big|^2\\
&+ \Big|\Big(A^{K^-}(^3S_1,-1) - \sqrt{1/2}A^{K^-}(^3P_1,-1)\Big)\sin\theta_p\\
&+ A^{K^-}(^3P_2,-1)(3\sin\theta_p\cos\theta_p)/2\Big|^2,
\end{aligned} \tag{9}$$

where the ϕ_p dependence goes away from the squared amplitudes of definite helicity λ. Before going further, we point out that the s-wave amplitudes alone cannot generate the asymmetric angular correlation for $B^- \to p\overline{p}K^-$ even though two s-wave amplitudes ($^1S_0$

and $^3S_1$) enter the right-hand side of Eq. (9): The reason is that the interference terms cancel out between $^1S_0$ and $^3S_1$ as

$$\begin{aligned}\frac{d\Gamma}{dm_{p\overline{p}}d\Omega_p}\bigg|_{B^-\to p\overline{p}K^-} &= \Gamma_0\Big[|A^{K^-}(^1S_0,0)+A^{K^-}(^3S_1,0)\cos\theta_p|^2 \\ &+ |-A^{K^-}(^1S_0,0)+A^{K^-}(^3S_1,0)\cos\theta_p|^2 \\ &+ |A^{K^-}(^3S_1,1)|^2\sin^2\theta_p+|A^{K^-}(^3S_1,-1)|^2\sin^2\theta_p\Big], \end{aligned} \tag{10}$$

and consequently $d\Gamma/d\Omega_p$ turns out to be symmetric under $\cos\theta_p\to-\cos\theta_p$. The same statement holds valid for p-waves alone. The observed steep asymmetry (Fig. 3) requires more than one orbital angular momentum, most likely s-wave and p-wave. It is very important experimentally to study how the angular correlation varies as p-waves increase with $m_{p\overline{p}}$ relative to s-waves across the threshold enhancement. It does not make sense to make a theoretical fit to the shape of the $m_{p\overline{p}}$ plot without large interference between different $p\overline{p}$ partial-waves.

The experimental uncertainty in the angular measurement limits quantitative analysis at present. Let us be content with qualitative analysis in this paper by approximating or interpreting for simplicity the angular correlation in Fig. 3 as $\sim(1+\cos\theta_p)^2$. This $\cos\theta_p$ dependence is realized if

$$A^{K^-}(^1S_0,0)\simeq A^{K^-}(^3S_1,0)\simeq A^{K^-}(^1P_1,0)\simeq A^{K^-}(^3P_0,0),\ \ (\mathrm{Exp}) \tag{11}$$

and all other amplitudes are negligible. An alternative solution is

$$A^{K^-}(^1S_0,0)\simeq -A^{K^-}(^3S_1,0)\simeq A^{K^-}(^1P_1,0)\simeq -A^{K^-}(^3P_0,0),\ \ (\mathrm{Exp}) \tag{12}$$

and all others are negligible. A small amount of $A^{K^-}(^3P_2,0)$ with the same sign as $A^{K^-}(^3S_1,0)$ would improve the fit a little by lowering the curve near $\cos\theta_p=0$ and raising it near $\cos\theta_p=\pm1$, but it is not crucial to the essence of our qualitative argument. While an accurate prediction is difficult because of our deficiency in knowledge of the quark distribution in baryons, the SD argument predicts, as we have argued above, the sign of slope opposite to Fig. 3: The angular dependence should be more like $(1-|a|\cos\theta_p)^2$ ($|a|\leq 1$) in the SD argument. This angular dependence corresponds to the partial-wave amplitudes,

$$A^{K^-}(^1S_0,0)\approx A^{K^-}(^3S_1,0)\approx -A^{K^-}(^1P_1,0)\approx -A^{K^-}(^3P_0,0),\ \ (\mathrm{SD}) \tag{13}$$

or alternatively,

$$A^{K^-}(^1S_0,0)\approx -A^{K^-}(^3S_1,0)\approx -A^{K^-}(^1P_1,0)\approx A^{K^-}(^3P_0,0),\ \ (\mathrm{SD}) \tag{14}$$

instead of Eqs. (11) or (12). Comparing the SD prediction with experiment, we find that the relative signs of the s-to-p-wave amplitudes are opposite. There are several alternatives that can alter the SD prediction in line with experiment: Sign reversal of the $^1S_0$ and $^3S_1$ amplitudes brings Eq. (13) to Eq. (11) and Eq. (14) to Eq. (12). Alternatively, sign reversal of $^1S_0$ and $^3P_0$ brings Eq. (13) to Eq. (12) and Eq. (14) to Eq. (11). Sign reversal of $^3S_1$ and $^1P_1$ also accomplishes the same. We ask what LD effect can possibly cause the sign reversal from Eq. (13) or (14) to Eq. (11) or (12). In the next section we argue that the desired sign reversal may occur if bound states exist in some of the $p\overline{p}$ channels. Unlike the argument that has ruled out a $p\overline{p}$ resonance, this is speculative and admittedly less clean part of our argument.

III. FINAL-STATE INTERACTION

The three-body final-state interaction FSI was analysed in the approximation of sum of two-body FSI since going beyond is mathematically formidable[17]. Fortunately, in the particle configuration of our interest where the invariant mass of $p\overline{p}$ is small and the K meson recoils fast against $p\overline{p}$, it is a good approximation and at least a common practice to separate the two-body FSI of $p\overline{p}$ ignoring the rest of FSI. Inclusion of $p\overline{p}$ annihilation channels is more a difficult problem. If one wants to make a quantitative analysis, this will be a main source of uncertainty.[3] Our task here is not to obtain numerically accurate results but to search a possible cause of sign flip for the amplitudes in the FSI. In order to make the sign flip argument plausible, we do not need much more than a basic argument of the elastic two-body FSI and its diagrammatic explanation.

The standard practice in FSI resorts to potential theory and incorporates FSI by modifying the decay amplitudes with final particle rescattering as[21]

$$A^{JI}(s) \to A^{JI}(s)/f^{JI}(-k), \tag{15}$$

where $f^{JI}(k)$ stands for the Jost function[22] of a partial-wave eigenchannel in variable $k = \frac{1}{2}\sqrt{s - 4m_N^2}$ $(s = (p_p + p_{\overline{p}})^2)$. It is normalized to $f^{JI}(\infty) = 1$. This FSI factor sums up ladders or bubbles of final particle rescattering in potential. The Jost function can be expressed with the phase of scattering amplitude δ^{JI} in the Omnés representation[23];

$$\begin{aligned} \frac{1}{f^{JI}(-k)} &= e^{\Delta^{JI}(\nu)}, \\ \Delta^{JI}(\nu) &= \frac{1}{\pi}\int_{\nu_0}^{\infty} \frac{\delta^{JI}(\nu')}{\nu' - \nu - i\epsilon} d\nu', \\ &= \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} \frac{\delta^{JI}(\nu')}{\nu' - \nu - i\epsilon} d\nu' + i\delta^{JI}(\nu), \end{aligned} \tag{16}$$

where $\nu = k^2$. The lower bound ν_0 of the dispersion integral is extended to the negative region $(s < 4m_N^2)$ when $p\overline{p}$ annihilation into meson channels is taken into account.

If annihilation and inelastic scattering are ignored, the phase $\delta^{JI}(\nu)$ would be equal to the phase shift of $N\overline{N}$ scattering according to the so-called Watson's theorem[24]. If there is a resonance in this elastic case, the phase shift $\delta^{JI}(\nu)$ rises from zero, passes through $\pi/2$ at the resonance $(\nu = \nu_R)$ and approaches π as $\nu \to \infty$. (Fig. 4.) Therefore the phase of the decay amplitude acquires a minus sign $(= e^{i\pi})$ above the resonance $\nu = \nu_R$.

This negative sign is easily understood in diagram. When a final particle pair is produced through a resonance, as depicted in Fig. 5 for $B^- \to p\overline{p}K^-$, the decay amplitude near the resonance takes the form of

$$A^{JI}(s) \simeq \overline{A}^{JI}(s)\frac{g^2(s)}{m_R^2 - im_R\Gamma_R(s) - s}, \tag{17}$$

where $\overline{A}^{JI}(s)$ is the amplitude in the absence of a resonance, m_R and $\Gamma_R(m_R^2)$ are the resonance mass and width, and $g^2(s)$ is *positive* at $s = m_R^2$. The reason for positivity of

[3] The FSI of the $p\overline{p}$ was recently studied for an enhancement in $J/\Psi \to \gamma p\overline{p}$[18–20]. In this process, $m_{p\overline{p}}$ is even closer to the threshold and consequently the coulombic FSI may be relevant.

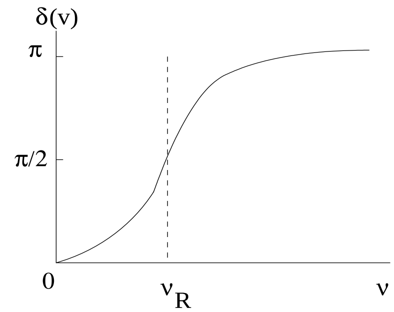

FIG. 4: The phase $\delta(\nu)$ of the FSI factor $e^{\Delta^{JI}}$ across an elastic resonance.

$g^2(m_R^2)$ is as follows: By the phase theorem of FSI[24], it holds that $\arg[A^{JI}(s)/\overline{A}^{JI}(s)] = \delta^{JI}(s)$. Therefore, the phase of $A^{JI}(s)/\overline{A}^{JI}(s)$ must be equal to $+\pi/2$, not $-\pi/2$, at the resonance peak. The phase of the resonant FSI factor in Eq. (17) must agree with this value at the resonance. The phase of $\arg[ig^2/m_R\Gamma(m_R^2)]$ is $+\pi/2$ at $s = m_R^2$ in agreement with the phase theorem if $g^2(m_R^2) > 0$. The function $g^2(s)$ is expected to be only mildly energy dependent even near the threshold since the centrifugal factor k^l of orbital motion resides in $A^{JI}(s)$ and $\overline{A}^{JI}(s)$, not in $g^2(s)$. Since $g^2(s)$ ($\simeq g^2(m_R^2)$) is positive, the sign of $A^{JI}(s)$ is the same as that of $\overline{A}^{JI}(s)$ below the resonance ($m_R^2 - s \gg m_R\Gamma_R$), but turns opposite above the resonance ($s - m_R^2 \gg m_R\Gamma_R$). This is another way of seeing the sign and energy dependence of the FSI factor across a resonance.

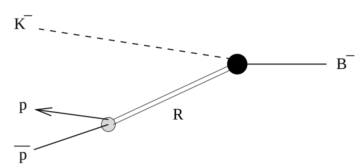

FIG. 5: Resonant production of $p\overline{p}$ in $B^- \to p\overline{p}K^-$.

This simple argument is modified by inelasticity above the $N\overline{N}\pi$ threshold and by annihilation into meson channels. Above the energies where inelastic channels start contributing substantially at $\sqrt{s} > 2m_N + m_\pi$, the FSI formulas of potential theory is no longer applicable. If we simply truncate the phase integral in Eq. (16) at $\nu = \nu_{\max}$ somewhere above the inelastic threshold, the FSI factor computed in the narrow-width (step-function) approximation turns out to be

$$e^{\Delta^{JI}(\nu)} \simeq \frac{\nu_{\max} - \nu}{\nu_R - \nu}, \quad (\nu \gg \nu_R), \tag{18}$$

which satisfies $f(-k) \to 1$ as $\nu(= k^2) \to \infty$. This FSI factor is negative between the resonance and the inelastic threshold;

$$1/f(-k) = e^{\Delta^{JI}(\nu)} < 0, \quad (\nu_R < \nu < \nu_{\max}). \tag{19}$$

It means that the FSI factor gives a minus sign above the resonance until energy goes up so high that inelasticity becomes important. While the negative sign is easy to understand, magnitude of the FSI factor is harder to estimate since it depends on the dispersion integral over the entire energy range.

Let us turn to the effect of the annihilation channels into mesons. The first issue is that the phase δ^{JI} of the decay amplitude is no longer equal to the phase of the $N\overline{N}$ scattering amplitude at any energy where annihilation occurs. An approximate equality between two phases holds only in those eigenchannels in which the channel coupling is weak between $N\overline{N}$ and the meson channels. It is not obvious whether this is the case for the relevant $N\overline{N}$ channels near the threshold. We must assume it here. Discussion will be made on this point below and in the next section. The other issue is whether relevant $N\overline{N}$ resonances really exist or not. The candidates of $N\overline{N}$ bound states and resonances indeed exist. Since the $N\overline{N}$ bound states can be only loosely bound, N and $\overline{N}$ are spatially separated outside the range of annihilation interaction and therefore the annihilation into mesons is suppressed. Meanwhile, being a bound state, the state cannot decay into a nucleon and an antinucleon since its mass m_B is below the $N\overline{N}$ threshold ($m_B < 2m_N$). However, there is an escape from this argument: The finite lifetime due to meson annihilation generates a width to the mass of the bound-state by the time-energy uncertainty. If this width is a little wider than the binding energy $\Delta = 2m_N - m_B$, the $N\overline{N}$ "bound state" can decay into $N\overline{N}$ (Fig. 6). This decay suffers a severe phase space suppression. When the $N\overline{N}$ bound state is produced "on mass shell" ($2m_N < M_{N\overline{N}} < 2m_N + \Gamma$ with Γ being the width), its decay branching fraction to the $N\overline{N}$ channel is small even if its coupling to $N\overline{N}$ is strong, *i.e.,* even if $g^2(s)$ in Eq. (17) is large. Consequently, such an $N\overline{N}$ bound state would appear as a relatively narrow meson resonance. On the other hand, when an $N\overline{N}$ pair is produced above the width of the bound state, the bound state can still enhance $N\overline{N}$ production through the small denominator of the resonance propagator. In experiment the transition from "on-shell" to "off-shell" occurs continuously above the threshold. The phase-space factor pushes the enhancement peak upwards from $2m_N$ to $m_{N\overline{N}} = 2m_N + O(\sqrt{\Delta^2 + \Gamma^2/4})$. When the phase-space factor is removed, the yield curve is expected to behave like $g^2(s)/[(m_{N\overline{N}} - m_B)^2 + \Gamma^2/4]$ at $m_{N\overline{N}} > 2m_N$. The BES Collaboration[25] first extracted the resonance parameters on this assumption when analysis was made only above the $p\overline{p}$ threshold in $J/\psi \to p\overline{p}\gamma$. In the three-body baryonic B decay, the events of small $m_{N\overline{N}}$ receive the SD enhancement, as we have argued. The same SD effect would be less prominent in J/ψ decay since the phase space is much smaller. Consequently the location and the shape of the enhancement may not be identical in B and J/ψ decays. Magnitude of the net $p\overline{p}$ enhancement is also dependent on dynamical environment of production. Despite such dynamical uncertainties we are fairly confident that if enhancement indeed occurs in the region of $m_{N\overline{N}} > 2m_N$, and if it couples to a state below it, the decay amplitude acquires the negative sign of FSI according to the diagram in Fig. 5 and the discussion following Eq. (17).

We remark on the coupling between $N\overline{N}$ and the annihilation channels. In the $p\overline{p}$ reaction at the threshold the annihilation cross section is larger than the elastic scattering cross section. Can it be compatible with weak coupling between $N\overline{N}$ and annihilation channels ? We should first note that the annihilation cross sections fall very rapidly with the inverse flux factor $1/|\mathbf{v}_p - \mathbf{v}_{\overline{p}}|$ above the threshold according to the "$1/v$" law of the exothermic reactions. We should also note that the large annihilation cross section is largely due to multitude of multi-meson annihilation channels with many different partial waves of sub-channels. In contrast, the elastic cross section near the threshold is almost entirely due to s-wave scattering. The annihilation cross section may not be so large in many partial-wave eigenchannels a little above the threshold. Therefore, the experimentally observed large

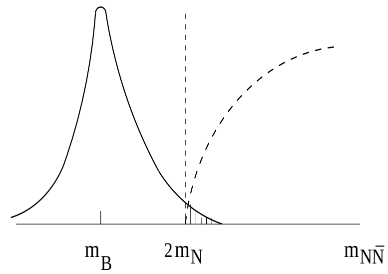

FIG. 6: The $N\overline{N}$ bound state X with mass m_B acquires a small width by annihilation decay. When the X is produced "on mass shell", only the upper corner (the hatched region) above the $N\overline{N}$ threshold contributes to the decay $X \to N\overline{N}$. The broken curve above $2m_N$ indicates the s-wave phase space. The $N\overline{N}$ production above the X mass shell can still be significant by the enhancement due to the X propagator.

total annihilation cross section is not an outright contradiction with weak coupling between $N\overline{N}$ and meson channels.

Theorists are not unanimous about existence of the $N\overline{N}$ bound states and resonances[18–20]. The recent discovery of the state $X(1835)$ in the radiative J/ψ decay suggests that an $N\overline{N}$ bound state may exist after all. If $X(1835)$ is indeed an $N\overline{N}$ bound state in ${}^1S_0$ or ${}^3P_0$, it is conceivable that a $N\overline{N}$ bound state exists in the ${}^3S_1$, ${}^3P_1$ or ${}^1P_1$ channel as well. Because of negative charge parity, experimental search is harder for ${}^3S_1$ and ${}^1P_1$ in the radiative J/ψ decay than search of ${}^1S_0$ and ${}^3P_J$. Leaving existence of $N\overline{N}$ bound states as an experimental issue still open, we proceed with our hypothesis of the sign flip and study the consequences in the pK^- angular correlation.

IV. $N\overline{N}$ BOUND STATES AND $p\overline{K}$ ANGULAR CORRELATION

The $\pi\pi\eta'$ resonance $X(1835)$ is the best candidate for the $N\overline{N}$ bound state. The sharp $p\overline{p}$ threshold enhancement observed in $J\psi \to p\overline{p}\gamma$ first hinted its existence as an $N\overline{N}$ bound state[25]. The mass was deduced at $1859^{+3}_{-10}(\mathrm{stat})^{+5}_{-25}(\mathrm{sys})$ MeV with width < 30 MeV. These values are sensitive to the method and assumptions involved in extracting them, *e.g.* rescattering and nonresonant background. They called it $X(1859)$. Two years later the BES Collaboration[13] identified a resonance in the $\pi\pi\eta$ mass plot and called it $X(1835)$, which is presumably the same state as $X(1859)$. It is most likely a state of ${}^1S_0$ with $I = 0$[13]. Assignment to ${}^3P_0$ (σ and η' in p-wave) of $I = 0$ cannot be excluded purely experimentally though less likely in theory because of the centrifugal repulsion. The width $(67.7 \pm 20.3 \pm 7.7)$ MeV is fairly narrow for its high mass. The upper tail of the width extends beyond the $p\overline{p}$ threshold and contributes to the decay into $p\overline{p}$. The BES Collaboration quotes the ratio of branching fractions as $\mathrm{Br}(X(1835) \to p\overline{p})/\mathrm{Br}(X(1835) \to \pi^+\pi^-\eta') \simeq 1/3$. In view of the tiny $p\overline{p}$ phase space, we reason that coupling of $X(1835)$ to $p\overline{p}$ is much stronger than that to mesons. For this reason the BES Collaboration suggests that $X(1835)$ is a likely candidate for a molecular or deuteron-like $N\overline{N}$ bound state. Such a bound state can play a dominant role in producing a $p\overline{p}$ pair in its eigenchannel near the threshold with little annihilation into mesons. This is exactly the state that we want for the cause of the sign flip.

If an $N\overline{N}$ bound state exists in ${}^1S_0$, a bound state may exist in ${}^3S_1$ as well by the property of the meson-exchange force between N and $\overline{N}$. If so, the decay amplitudes $A^{K^-}({}^1S_0, 0)$ and

$A^{K^-}({}^3S_1, 0)$ flip their signs from the SD ones in Eq. (13) or Eq. (14) to the experimentally observed ones in Eq. (11) or Eq. (12). In this way we would have a chance to obtain the observed trend $(1+\cos\theta_p)^2$ for the pK^- angular correlation. Since the $B^- \to p\overline{p}K^-$ amplitudes consist of both $I=0$ and $I=1$ of $N\overline{N}$, the sign flip would occur most effectively when the $I=0$ amplitudes dominate over the $I=1$ amplitudes;

$$|A^0({}^1S_0, 0)| \gg |A^1({}^1S_0, 0)|, \quad |A^0({}^3S_1, 0)| \gg |A^1({}^3S_1, 0)|. \tag{20}$$

If the ${}^3S_1$ bound state is in $I=1$ instead of $I=0$, the second inequality in Eq. (20) should be reversed in direction.

This is our proposal for the resolution of the angular correlation puzzle. As was mentioned below Eq. (14), there are other possibilities if a p-wave bound state exists. In those cases the s-wave $N\overline{N}$ bound state should exist only in ${}^1S_0$ or ${}^3S_1$, not in both. The spin splitting is generally weaker in nuclear forces than the orbital-angular-momentum splitting. If this prevails in the $N\overline{N}$ force, the bound states should appear first in the s-wave channels and then in the p-wave channels. However, we should keep our mind open to the other possibilities of sign reversal in the p-wave amplitudes. In order to make further advance, we need to know more about $X(1835)$ and to search for more candidates of the $N\overline{N}$ bound states. In experiment of B meson physics, we are anxious to know the $m_{p\overline{p}}$ dependence of the angular correlation since it will provide important pieces of information about spin-parity, mass and isospin of the bound states.

As for $B^0/\overline{B}^0 \to p\overline{p}K_S$, the measurement was made for the oscillating B^0-$\overline{B}^0$ averaged over time and therefore no flavor information is available[11]. So long as the penguin interaction dominates, the time-averaged pK_S angular distribution is symmetric under $\cos\theta_p \to -\cos\theta_p$ in general. (cf Eq. (3).) Specifically, if we keep only the amplitudes of $J \leq 1$ and $\lambda = 0$ in Eq. (9), the angular distribution is

$$\begin{aligned} \frac{d\Gamma}{dm_{p\overline{p}}d\Omega_p}\bigg|_{B^0/\overline{B}^0\to p\overline{p}K_S} &= \Gamma_0\Big[\Big(|A^{\overline{K}^0}({}^1S_0, 0)|^2 + |A^{\overline{K}^0}({}^3P_0, 0)|^2\Big) \\ &+ \Big(|A^{\overline{K}^0}({}^3S_1, 0)|^2 + |A^{\overline{K}^0}({}^1P_1, 0)|^2\Big)\cos^2\theta_p\Big]. \end{aligned} \tag{21}$$

The curve of this angular correlation for pK_S is concave in $\cos\theta_p$. The data[11] in Fig. 7 indeed show the tendency of roughly $\sim 1+\cos^2\theta_p$ albeit with very large uncertainty. The branching fraction was also measured[11] and its ratio to that of $p\overline{p}K^-$ is

$$\mathrm{Br}(B^0/\overline{B}^0 \to p\overline{p}K_S)/\mathrm{Br}(B^+ \to p\overline{p}K^+) \simeq 0.23 \tag{22}$$

with roughly $\pm 20\%$ of statistical errors and $\pm 10\%$ of systematic errors. This number would be 0.5 if the $I=0$ amplitudes completely dominates over the $I=1$ amplitudes in Eq. (20). If the I=1 amplitudes are about 20% of the $I=0$ amplitudes, however, this ratio 0.23 can be reproduced.

V. $\Lambda\overline{p}$ CHANNEL

The threshold enhancement has been observed in other three-body baryonic final states, $\Lambda\overline{p}\pi^+$, $p\overline{p}\pi^-$, and $\Lambda\overline{\Lambda}K^+$ (charge conjugated states combined) as well as in $\Lambda\overline{p}\gamma$ and many

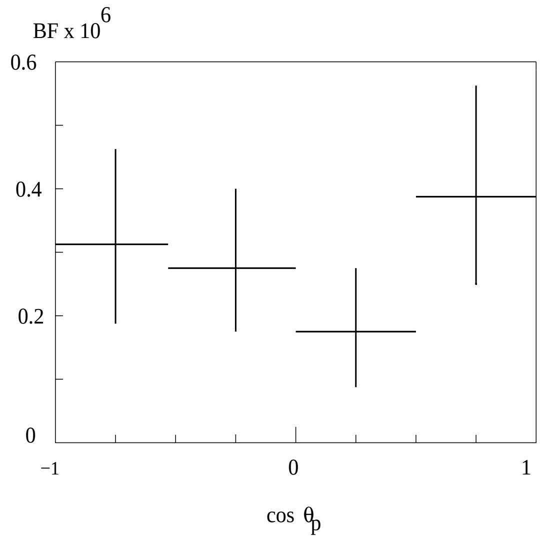

FIG. 7: The pK_S angular correlation in the $p\overline{p}$ rest frame of $B^0/\overline{B}^0 \to p\overline{p}K_S$ [Ref. 10].

decay modes of the $b \to c$ transition. However, the angular correlation has been measured only for a few of them; $\Lambda\overline{p}\pi^+$, $\Lambda\overline{p}\gamma$, and $\Lambda_c\overline{p}\pi^+$ all in low statistics. Let us look into the $\Lambda\overline{p}$ enhancement observed in $\Lambda\overline{p}\pi^+$ and $\Lambda\overline{p}\gamma$ of $\overline{B}^0$ decay and the conjugate, for which the dominant process is the penguin decay.

In the color-dominant process of $\overline{B}^0 \to \Lambda\overline{p}\pi^+$ the spectator $\overline{d}$-quark forms π^+ through capture by the energetic u-quark that comes directly from the strong penguin interaction. Since neither Λ nor $\overline{p}$ picks up the slow spectator, their average energies in the $\overline{B}^0$ rest frame should be comparable. That is, the $\overline{p}\pi^+$ angular correlation in the $\Lambda\overline{p}$ rest frame ought to be more or less symmetric and flat in $\cos\theta_{\overline{p}}$. This naive SD prediction is in line with experiment within large uncertainty: The measured angular correlation[11] does not show marked asymmetry nor large variation in $\cos\theta_{\overline{p}}$. (See Fig. 8.) The $\overline{p}\pi^+$ angular distribution can be fitted with any of 1, $1 + |b|^2\sin^2\theta_{\overline{p}}$, and $(1 - |b|\cos\theta_{\overline{p}})^2$ with small constant $|b|$. Although the angular distribution is consistent with the SD prediction, one cannot rule out a resonance for the $\Lambda\overline{p}$ enhancement: A resonance of $J = 0$ leading to $d\Gamma/d\Omega_{\overline{p}} \sim 1$ is certainly acceptable. A resonance of $J = 1$ is neither ruled out since the flat distribution arises with $|A^1_{++;0}|^2 + |A^1_{--;0}|^2 \approx \frac{1}{2}(|A^1_{+-;0}|^2 + |A^1_{-+;0}|^2)$.

Let us turn to $\Lambda\overline{p}\gamma$. In the SD picture the energetic s-quark is emitted against γ by the radiative penguin interaction and becomes the constituent of Λ. Other quarks and antiquarks are produced through strong interaction nearly collinearly against γ to avoid creating a fat gluon. Since the spectator $\overline{d}$ enters in $\overline{p}$, the antiproton is less energetic in the rest frame of $\overline{B}^0$ than Λ. The SD prediction is therefore that $\overline{p}$ moves along γ in the $\Lambda\overline{p}$ rest frame, that is, $d\Gamma/d\Omega_{\overline{p}}$ should rise toward $\cos\theta_{\overline{p}} = 1$, where $\theta_{\overline{p}}$ is the angle between $\overline{p}$ and γ in the $\Lambda\overline{p}$ rest frame. The observed $\overline{p}\gamma$ angular correlation clearly shows this trend in line with the SD prediction. (See Fig. 9.)

Although the SD prediction is right for $\Lambda\gamma$, a resonance cannot be ruled out for $\Lambda\overline{p}$. The helicity expansion of the $\Lambda\overline{p}\gamma$ amplitude is modified by spin of γ. The helicity of γ is -1 since the s-quark emitted by the penguin interaction is left-handed. Therefore the component of

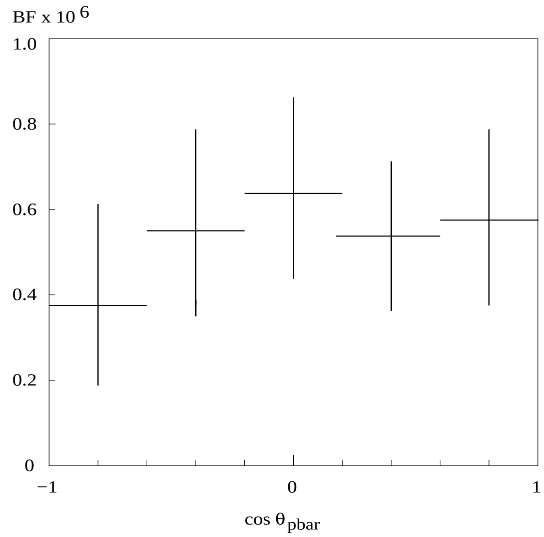

FIG. 8: The $\overline{p}\pi^+$ angular correlation in the $\Lambda\overline{p}$ rest frame of $\overline{B}^0 \to \Lambda\overline{p}\pi^+$ [Ref. 10].

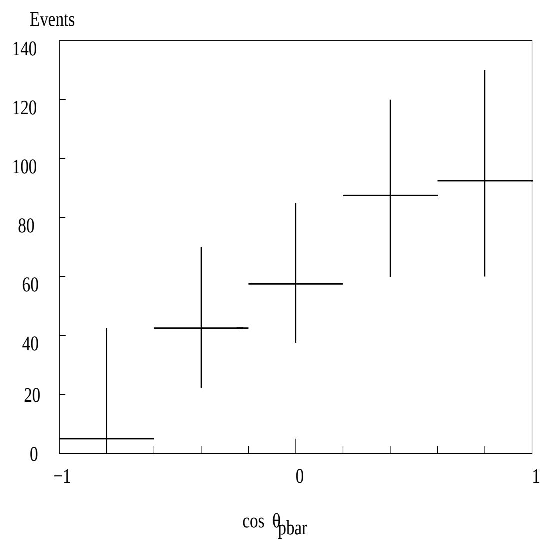

FIG. 9: The $\overline{p}\gamma$ angular correlation in the $\Lambda\overline{p}$ rest frame of $B^- \to \Lambda\overline{p}\gamma$ [Ref. 26].

total angular momentum along the photon momentum $\mathbf{J}\cdot\hat{\mathbf{p}}_\gamma$ is $+1$ for $\Lambda\overline{p}$ in their rest frame. The $\Lambda\overline{p}$ angular correlation is given generally by

$$\left.\frac{d\Gamma}{dm_{\Lambda\overline{p}}d\Omega_p}\right|_{B^-\to\Lambda\overline{p}\gamma} = \Gamma_0 \sum_{\lambda_\Lambda\lambda_{\overline{p}}} |\sum_J A^J_{\lambda_{\overline{p}}\lambda_\Lambda;1} d^J_{1\lambda}(\theta_{\overline{p}})|^2, \quad (\lambda = \lambda_{\overline{p}} - \lambda_\Lambda). \tag{23}$$

Note here that the first subscript of $d^J_{1\lambda}(\theta_{\overline{p}})$ is 1 owing to $J_z = +1$. The $\Lambda\overline{p}$ enhancement cannot be a resonance of $J = 0$ since $J \geq |\mu|$ for $d^J_{\mu\lambda}(\theta_{\overline{p}})$. However, we cannot rule out a $J = 1$ resonance since with

$$1 + \cos\theta_{\overline{p}} = 2d^1_{11}(\theta_p)^2 + d^1_{10}(\theta_p)^2, \tag{24}$$

the linear correlation $\sim 1 + \cos\theta_{\overline{p}}$ arises if the helicity amplitudes for $\Lambda\overline{p}\gamma$ happen to obey, for instance,

$$|A^1_{+-;1}|^2 \approx 2\Big(|A^1_{++;1}|^2 + |A^1_{--;1}|^2\Big), \quad A^1_{-+;1} \approx 0. \tag{25}$$

Because of the preferred photon helicity, the $\overline{p}\gamma$ angular correlation can be asymmetric under $\cos\theta_{\overline{p}} \to -\cos\theta_{\overline{p}}$ even with a single partial wave unlike those of pK^- and $\overline{p}\pi^+$. Combining our observations in $B^- \to \Lambda\overline{p}\pi^+$ and $B^- \to \Lambda\overline{p}\gamma$ together, we can rule out a $J = 0$ resonance for $\Lambda\overline{p}$, but not a $J = 1$ resonance. However, there is no motivation to call for a $\Lambda\overline{p}$ resonance or bound state at present until we see clear discrepancy with the SD prediction.

VI. SUMMARY AND REMARKS

Three-body baryonic decay modes are favoured over two-body baryonic decay modes since a baryon-antibaryon pair may be emitted nearly collinearly in three-body decay. Indeed experiment confirms that the invariant mass of the baryon-antibaryon pair is strongly enhanced near the threshold in most modes. Although the SD picture appears to describe general trends of three-body decays in most cases, we have encountered one clear contradiction with the SD picture in the angular correlation between p and $K^\pm$ in $B^\pm \to p\overline{p}K^\pm$.

Failure of the SD picture means that some LD effect enters the process of $B^- \to p\overline{p}K^-$ and reverses the angular dependence. We have pointed our finger to the nucleon-antinucleon bound states for the cause of sign flip of the SD amplitudes and have given a simple diagrammatic explanation for it. To explain the decay angular correlation for $B^+ \to p\overline{p}K^+$, we have postulated that $X(1835)$ be the $^1S_0$ bound state of $N\overline{N}$. That is, $X(1835)$ is a molecular six-quark state $qqq\overline{qqq}$ or a deuteron-like state and primarily couples to $N\overline{N}$ rather than to mesons. Many theorists have made the same or similar proposals on the nature of $X(1835)$, with motivations very different from ours. In addition, we need a $^3S_1$ bound state of $N\overline{N}$. The maximum asymmetry of the angular correlation should occur at the energy where kinematically rising p-wave amplitudes become comparable with the falling s-wave amplitudes. The $m_{p\overline{p}}$ angular dependence will tell us a lot about dynamics near the $p\overline{p}$ threshold.

Our argument depends on strong interaction dynamics near the $N\overline{N}$ threshold that has not been proven nor disproven experimentally. Some might feel that we have blown out a possible solution to a small puzzle into a farfetched speculation. We cannot counter such objections effectively. Our argument presented in this paper is a conjecture or a hypothesis, certainly not a theorem. Although our argument is only exploratory and speculative, the sign flip by a bound state or a resonance can occur generally and cause failure of the simple quark-gluon argument of multi-body B meson decay.

While our argument of the sign flip is exploratory in nature, we would like to emphasize that the partial-wave expansion analysed here will be very useful as a general tool to penetrate into complexity of three-body decay dynamics. To show its usefulness, we have ruled out convincingly the resonance hypothesis for the $p\overline{p}$ threshold enhancement. We have also shown that the FSI in s-waves or in p-waves alone should not describe the enhancement either. While the angular correlation measurement is not extensive nor accurate enough at present, we expect that partial-wave analysis of three-body decay will shed more of new light on dynamics of B decay in near future since the data are rapidly accumulating.

Acknowledgments

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